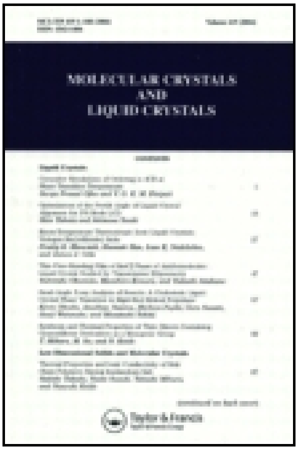

Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl20

Influence of the Multiple Beam Interference on Phase Retardation of Liquid Crystal Cells

Ivan Chernyaev [a], Anastasia Suvorova [a b], Fedor Podgornov [a b] & Wolfgang Haase [a]

[a] Eduard-Zintl-Institute for Inorganic and Physical Chemistry, Darmstadt University of Technology, Darmstadt, Germany

[b] Nonlinear optics laboratory, South Ural State University, Chelyabinsk, Russia

Version of record first published: 22 Sep 2010

Mol. Cryst. Liq. Cryst., Vol. 488, pp. 219–230, 2008

ISSN: 1542-1406 print/1563-5287 online
DOI: 10.1080/15421400802240672

Influence of the Multiple Beam Interference on Phase Retardation of Liquid Crystal Cells

Ivan Chernyaev[1], Anastasia Suvorova[1,2], Fedor Podgornov[1,2], and Wolfgang Haase[1]
[1]Eduard-Zintl-Institute for Inorganic and Physical Chemistry, Darmstadt University of Technology, Darmstadt, Germany
[2]Nonlinear optics laboratory, South Ural State University, Chelyabinsk, Russia

The aim of this article is to carry out theoretical investigation of the influence of the multiple beam interference on the phase retardation of liquid crystal cells. For the theoretical description of this effect, Heavens-Clarke matrix formalism was utilized. As a result, the significant influence of interference effects on the phase shift of the transmitted light is demonstrated. It is shown that the interference effects are determined by geometrical and optical parameters of the constituting layers of a liquid crystalline cell. Moreover, it is demonstrated that for a given liquid crystal at a given temperature an optimized combination of variable parameters as thicknesses of liquid crystal layer, polymer layer, electrode layer, glass substrate layer the influence of the multiple interference on the phase retardation can be minimized.

Keywords: liquid crystal cell; multi-beam interference; phase retardation

INTRODUCTION

Spatial light modulators are widely used in the modern optoelectronics (e.g. liquid crystal displays (LCDs), modulators, compensators, etc.). Nowadays, several phase-amplitude light modulation techniques exist. These techniques are usually based on modulation methods such

Authors are thankful to German Federal Ministry for Education and Research for financial support of our bilateral cooperation. Dr. F. Podgornov wants to acknowledge the financial support from NATO through the reintegration grant (CBP.NR. RIG.982107).

Address correspondence to Wolfgang Haase, Institut fur physikalische Chemie, AG Kondensierte Materie, Fachbereich Chemie, Technische Universitat Darmstadt, Petersenstrasse 20, Darmstadt, D-64287, Germany. E-mail: haase@chemie.tu-darmstadt.de

as mechanical rotation of anisotropic crystal wave plates, electro-optic Pockels and Kerr effects. But such technologies have in spite of high modulation accuracy and rate some significant disadvantages like high energy consumption, bulky form-factor, high cost. Such drawbacks can be remedied by using liquid crystal (LC) light modulators.

Spatiotemporal light modulation using liquid crystals is based on the change of the optical properties of the modulator due to reorientation of molecules under the action of the driving voltage. In order to estimate the LC cell phase retardation by a given applied voltage, the standard expression used is:

$$\Gamma = \frac{2\pi}{\lambda}(n_e - n_o)d \tag{1}$$

Here Γ – LC layer phase retardation, d – LC cell gap, λ – irradiation wavelength, n_o – ordinary refractive index of the media, n_e – extraordinary refractive index.

However if the coherence of the light source is rather high the interference occurs of the waves reflected by interfaces of the cell. In this case the estimation of the phase retardation with (1) leads only to approximate values.

Considering simple optical systems, the influence of the interference of multiple reflections on the phase modulation is widely described in the literature [1–3]. The nature of the interference depends both on the parameters of the optical system (characteristic dimensions, optical anisotropy) and the properties of the propagating radiation (state of polarization, wavelength).

The structure of the LC modulator is more complex. It represents an anisotropic stratified media with properties being dependent on the wavelength of the propagating radiation. In such a system partial waves can appear at all of the interfaces, hence interference effects might considerably influence the phase retardation of the LC modulator, i.e. LC cell.

Interference effects in LC cells were studied by several authors [4–8]. However, in every case only a simplified modeling of the LC cell was foreseen, namely in scope of the application in LCDs. In [4], the Fabry-Perot effect was investigated for LC cells. However while performing the calculations only the LC-polymer interface was taken into account. In [5] the polymer and the ITO electrode layers were not taken into account, but the optical properties of the glass – cholesteric LC – glass system were investigated. In [6] the influence of the interference effects on the contrast ratio of

LC devices with remaining birefringence compensation was studied. It was clearly demonstrated that the transmission of light through an ideally compensated device in a wide spectral range is influenced by inner reflections of the ordinary and extraordinary waves, however only the generation of partial waves at the air-LC interface was taken into consideration.

Interference effects created by multiple reflections were experimentally studied in [7–8]. The influence of interference effects on the accuracy of measurement of such parameters of LC cells as twist angle and anchoring energy was shown.

To the best of our knowledge interference effects based on multiple reflections in the whole multilayer structure of LC cell were never considered in order to estimate the phase retardation of LC cell at a given wavelength and cell parameters.

CALCULATION PROCEDURES FOR THE OPTICAL PROPERTIES

The propagation of the electromagnetic radiation in a media is usually described by the well known Maxwell equations. At the same time analytical solutions of the Maxwell equations can be found only in some specific cases. In order to describe the interaction of the radiation with an arbitrary inhomogeneous anisotropic media a number of different techniques were developed. A common feature for all of them is to minimize the number of variables which describe the electromagnetic wave. Some of the most popular methods are after Jones [9], Mueller [9] or Berreman [10].

The Mueller matrix method doesn't allow the description of the influence of the interference effects on the properties of the transmitted radiation. This is because this method operates with the intensity of the wave rather than its amplitude and phase. Drawback of the methods after Jones and Berreman is the comparative mathematical complexity to find out algorithms. For instance, in the Berreman method the exponent of the 4×4 matrix is required to be calculated. This is usually realized either approximately by expansion of the exponent in a Taylor series [11] or via exact solution by applying the Silvestre theorem for the representation of the matrix function as finite series [12]. In both cases a number of difficulties in the computer realization of the algorithms appears (dividing a media on a large number of layers with thicknesses much less than the radiation wavelength [11], solution of 0/0 uncertainty [12], etc.).

Interference effects can be taken into attention using the expanded Jones matrix formalism. Following this technique one needs to calculate a special coefficient describing the anisotropy of refractive indexes and the Fabry-Perot effect. However this is in case of anisotropic media a rather complex procedure.

A more simple approach was developed by Clarke *et al.* [2,13]. This approach is based on Heavens matrix formalism [14], which uses 2×2 matrices characterizing the interfaces of the compound optical system. By Heavens, an elementary matrix M_k describing a k-th interface and the antecedent media has the following form:

$$M_k = \begin{bmatrix} \frac{1}{t_k} e^{i\delta_{k-1}} & \frac{r_k}{t_k} e^{i\delta_{k-1}} \\ \frac{r_k}{t_k} e^{-i\delta_{k-1}} & \frac{1}{t_k} e^{-i\delta_{k-1}} \end{bmatrix}. \quad (2)$$

Here t_k – transmission coefficient of k-th interface, r_k – reflection coefficient of k-th interface, δ_{k-1} – phase retardaion of the media followed by the k-th interface.

In case of optically anisotropic systems there is a need to perform in a separate manner calculations for each orthogonal component of the electromagnetic wave. For doing this the following matrix was suggested by Clarke [13].

$$M_k = \begin{bmatrix} \frac{1}{t_\perp} e^{i\phi_\perp} & \frac{r_\perp}{t_\perp} e^{i\phi_\perp} & 0 & 0 \\ \frac{r_\perp}{t_\perp} e^{-i\phi_\perp} & \frac{1}{t_\perp} e^{-i\phi_\perp} & 0 & 0 \\ 0 & 0 & \frac{1}{t_\|} e^{i\phi_\|} & \frac{r_\|}{t_\|} e^{i\phi_\|} \\ 0 & 0 & \frac{r_\|}{t_\|} e^{-i\phi_\|} & \frac{1}{t_\|} e^{-i\phi_\|} \end{bmatrix}. \quad (3)$$

Here $\|$ and $\perp$ symbols correspond to the general optical axes of the media, Φ – the phase of the corresponding orthogonal component.

The matrix of the entire optical system can now be calculated by sequential multiplication of elementary matrices corresponding to each interface.

By Clarke [13], the electric field of the propagating electromagnetic wave through the media is described via the column:

$$\begin{bmatrix} \overrightarrow{E}_\perp \\ \overleftarrow{E}_\perp \\ \overrightarrow{E}_\| \\ \overleftarrow{E}_\| \end{bmatrix}, \quad (4)$$

$\rightarrow$ and $\leftarrow$ denote the direction of the orthogonal components of the wave propagating inside the media.

The transformation of the electric field wave for the entire system can be expressed by:

$$\begin{bmatrix} \overrightarrow{E}'_{\perp} \\ \overleftarrow{E}'_{\perp} \\ \overrightarrow{E}'_{\parallel} \\ \overleftarrow{E}'_{\parallel} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} \overrightarrow{E}_{\perp} \\ \overleftarrow{E}_{\perp} \\ \overrightarrow{E}_{\parallel} \\ \overleftarrow{E}_{\parallel} \end{bmatrix}. \tag{5}$$

Now, the phase retardation Γ and transmission T of the system can be easily extracted from matrix elements m_{11} and m_{33}:

$$\tan \Gamma_{\perp,\parallel} = \frac{\mathrm{Im}(T_{\perp,\parallel})}{\mathrm{Re}(T_{\perp,\parallel})}, \quad T_{\perp,\parallel} = \frac{\overrightarrow{E}_{\perp,\parallel}}{\overrightarrow{E}'_{\perp,\parallel}} = \frac{1}{m_{11,33}}. \tag{6}$$

Thus the Heavens-Clarke matrix formalism allows a simple and effective description of the propagation of the electromagnetic radiation through a stratified anisotropic optical system by taking into consideration the interference effects created by multiple reflections. At the same time one should note that this technique operates with an ideal plane monochromatic electromagnetic wave propagating straight perpendicularly to an ideal plane-parallel stratified media. The interplay with the radiation consists only in the phase shift between components of the wave.

MODEL OF LIQUID CRYSTAL LIGHT MODULATOR

In this work the light propagation through the standard "sandwich" cell has been calculated using Havens-Clarke formalism. Such cell has the following structure (Fig. 1).

The cell consists of two plane-parallel glass substrates covered by transparent electrodes and polymeric alignment layers. These substrates are cemented in such a way that some gap is formed between them. This gap is filled with the liquid crystal.

Parameters of the cell can be different depending on the application. Usually in LC displays the cell gap, i.e. liquid crystal layer thickness, varies between 3 and 10 μm, the substrate thickness between 0.7 to 1.1 mm, the transparent electrode thickness between 20 to 300 nm depending on the required impedance and transmittance. The polymeric aligning layer is generally about 50 nm thin. Depending on each applications, e.g. in polarimetry, telecommunication etc. the

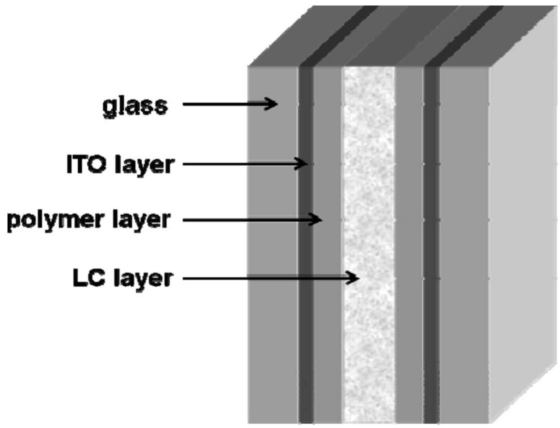

FIGURE 1 LC cell in schematic representation.

thickness of each individual layer can be considerably different from the above given ranges. Thus in order to obtain quarter-wave or half-wave phase retardation the cell gap can be up to 100 μm depending on the birefringence of the liquid crystal under application, the polymeric alignment layer up to 500 nm.

As for the material of the different cell elements, the substrates are generally out of K8 glass with a refractive index 1.51. The refractive index of the polymeric aligning layer depends on the chemical structure, but generally ranges between 1.5–1.6. Transparent electrodes are usually indium tin oxide (ITO). The refractive index of those depends significantly on the applied sputtering technology variable in a wide range from 1.7 to 2.2. As it is well known liquid crystals are optically anisotropic media with birefringence up to 0.5, sometimes up to about 0.7 [15]. All the magnitudes for the refractive indexes mentioned above are selected for 632.8 nm wavelength. It must be noted that only the real part of the refractive index is taken into account. This is reasonable only when the contribution of the absorbance to the polarimetric properties of the modulator can be neglected.

The dispersion of the refractive index of K8 glass, polymer alignment layer and ITO layer can be calculated using the well known Cauchy formula:

$$n^2 = \frac{A}{\lambda^2} + B, \tag{7}$$

A, B – constants, n – refractive index of the media, λ–irradiation wavelength.

TABLE 1 Properties of the Different Layers of the LC Cell

	Layer thickness, μm	Refractive index at 632.8 nm wavelength
Glass substrates	100–1000	1.51
ITO electrodes	0.01–0.3	1.92
Polymer alignment layers	0.05–0.5	1.67
Liquid crystal 5CB (25°C)	1–10	ordinary: 1.53 extraordinary: 1.71

In order to determine A and B, the refractive indexes for two different wavelengths must be known. K8 glass refractive index was taken from hand-book of optical materials, for polymer alignment layer from [16], for ITO from [17]. As for liquid crystals, it has been demonstrated by [18] that the standard formula written above cannot be used. Therefore the single-band dispersion model after [18] which perfectly fits to the experimental data is utilized in this work. The corresponding expressions for the ordinary and extraordinary refractive indexes are the following:

$$n_e \cong 1 + g_{\parallel} \frac{\lambda^2 \lambda_{\parallel}^2}{\lambda^2 - \lambda_{\parallel}^2}, \quad n_o \cong 1 + g_{\perp} \frac{\lambda^2 \lambda_{\perp}^2}{\lambda^2 - \lambda_{\perp}^2} \tag{8}$$

n_e and n_o–extraordinary and ordinary refractive indexes, $g_{\parallel,\perp}$ and $\lambda_{\parallel,\perp}$ are constants. In this work refractive indexes of the nematic liquid crystal 5CB at room temperature (25°C) given in [18] are used in the calculations.

All the LC cell parameters which were used for the simulation of the light propagation are given in Table 1.

RESULTS

In order to calculate the phase retardation for light transmitted through the LC cell at a given wavelength and cell parameters, the software using Heavens-Clarke matrix formalism was utilized.

Figure 2, left, shows the calculated phase retardation versus wavelength in the visible range with and without interference. It′s obvious that interference effects contribute significantly to the magnitude of the phase retardation. Figure 2, right, displays the same dependences in the narrow wavelength range (nearby helium-neon laser emission wavelength). It can be noted that relatively large oscillations in frequency and amplitude take place.

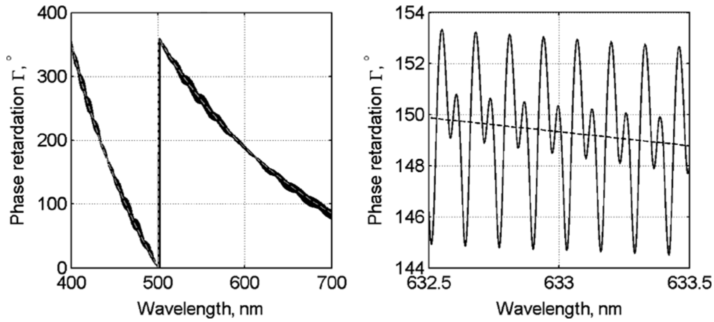

FIGURE 2 Dispersion of the LC cell phase retardation with (full line) and without (dashed line) interference effects in the whole multilayer structure of the cell. Left: visible range; right: close to helium-neon laser wavelength. Glass substrate thickness 1 mm, ITO layer 20 nm, polymer 200 nm, LC layer 5 μm.

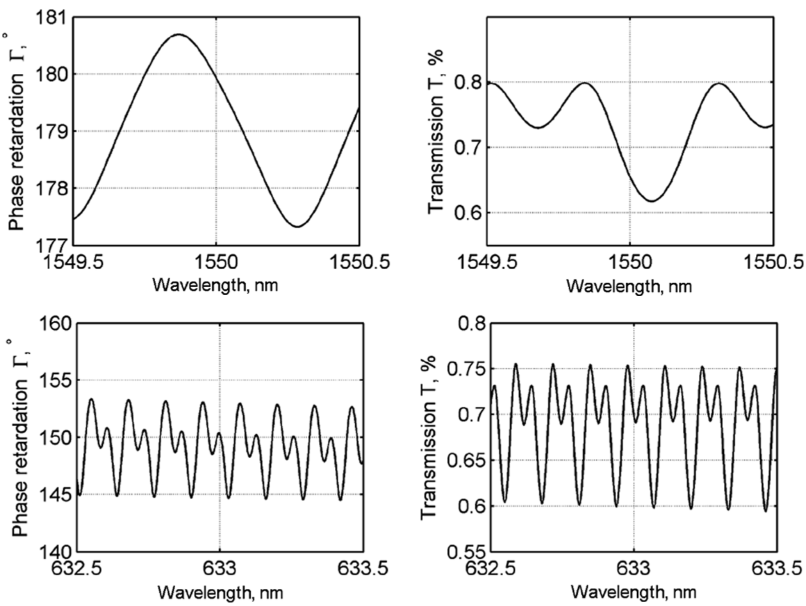

FIGURE 3 Phase retardation and transmission of the LC cell for two different wavelength ranges including interference effects in the whole multilayer structure. Glass substrate thickness 1 mm, ITO layer 20 nm, polymer 200 nm, LC 5 μm.

Phase retardation and dispersion of the transmission of the LC cell between crossed polarizers are presented in Figure 3.

The transmission T through the LC cell was calculated using the well known expression:

$$T = \frac{1}{2}(a_o^2 + a_e^2 - 2a_o a_e \cos\Gamma) \tag{9}$$

Here a_o and a_e – amplitudes of ordinary and extraordinary components of radiation transferred trough the cell, Γ – phase shift between those components (intensity of the incident radiation is unitary, polarizers are oriented at 45° to LC cell optical axes).

It can be well seen that the interference effects significantly influence the intensity of the transmitted radiation by up to 15%, furthermore the oscillation frequency of the transmittance is the same as for the phase retardation. Remarkable, in the infrared range the oscillations of phase retardation and transmission are less frequent compared to those in the visible range.

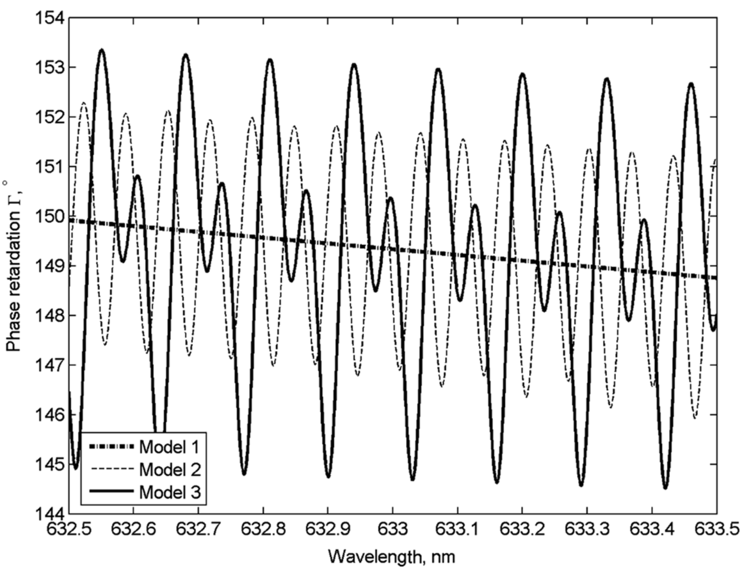

FIGURE 4 Dispersion of the LC cell phase retardation vs. wavelenght nearby the helium-neon laser line. Model 1: glass – LC – glass. Model 2: air – glass – LC – glass – air. Model 3: air – glass – ITO – polymer – LC – polymer – ITO – glass – air. Glass substrate thickness 1 mm, ITO layer 20 nm, polymer 200 nm, LC 5 μm.

In order to estimate the contribution of each layer of the LC cell to the interference, several models allowing the simulation of the LC cell were considered. In the first model only multiple reflections at the LC-glass interface are taken into account, in the second–LC-glass and glass-air (vacuum), in the third – all the LC cell interfaces. Figure 4 depicts the result of the simulations.

It is evident that in order to find exact values for the phase retardation at a given wavelength one cannot neglect one or another LC cell structural element. Each element makes its own contribution to the interference effects. In particular, the influence of ITO electrodes and polymer alignment layers, model 3, is visible by additional low-amplitude oscillations at the same frequency. It can be also seen that the high frequency in the oscillations has its origin in multiple reflections at the glass-air interface. This can be explained by the fact that the thickness of the glass substrate exceeds the radiation wavelength, which leads to rather large phase shift of the partial waves.

Depending on the application and the fabrication technology the thickness of all layers of the LC cell can be different from that assumed within the above models. Moreover variation of the thickness

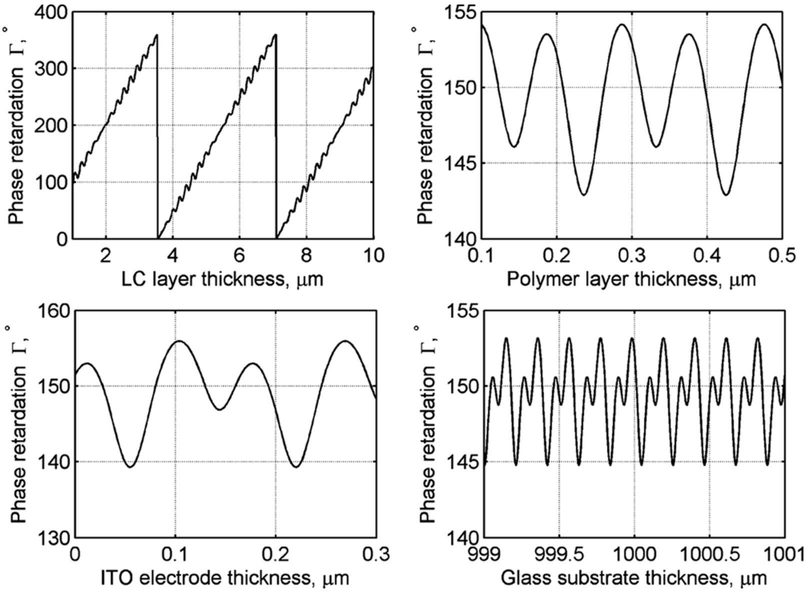

FIGURE 5 LC cell phase retardation vs. thickness of each single layer by holding thicknesses of the remaining layers constant (thicknesses of unvaried parameters: glass 1 mm, electrodes 20 nm, polymer 200 nm, LC 5 μm). Interference effects in the whole multilayer structure are included.

of the layers in the entire LC cell can always take place. Figure 5 presents the phase retardation of the LC cell versus the thickness of each different layer at the wavelength of the helium-neon laser (632.8 nm). Interference of multiple reflections at all interfaces in the LC cell was allowed. It is clear from Figure 5, the thickness of the LC layer influences generally the absolute magnitude of the phase retardation, the influence of thickness variations of the ITO electrodes and the polymer alignment layers is relatively low. However one can clearly seen from Figure 5 there is a considerable dependence of the phase retardation on the glass thickness. Every tiny thickness deviation can lead to significant changes in the phase retardation. Figure 5 top left also shows that in the region of $\lambda/4$ the fluctuations of the phase retardation are remarkable, whereas for the given parameter set at $\lambda/2$ the fluctuations in the phase retardation are low.

For practical application, e.g. for preparing $\lambda/4$ or $\lambda/2$ cells, one is interested to minimize the influence of interference of multiple reflections at the various interfaces on the phase retardation or better to set down it completely. This can be done by varying the thickness of layers of the LC cell. In Figure 6 conditions are selected in order to cancel out the fluctuations to a good extent. This figure can be compared with Figure 2 right. Obviously a partial compensation of the influence of multiple interference takes place. It should be noted that one may vary not only the thickness of the layers of the LC cell but also the refractive indexes of polymer and ITO layers for further compensation of the contribution due to interference effects.

Further studies are needed to verify results of our calculations under experimental conditions.

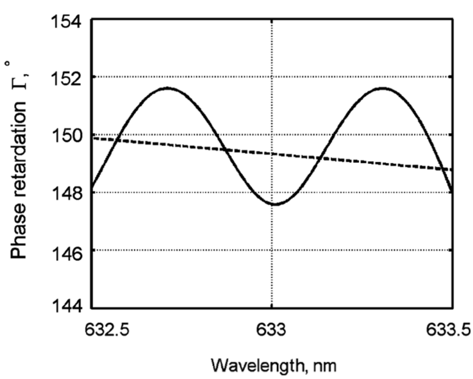

FIGURE 6 Phase retardation of LC cell vs. wavelength for nearly optimized parameters of LC cell (glass substrates 0.1 mm, ITO electrodes 20 nm, polymer 150 nm, LC layer 5 μm) with interference effects (full line) and without (dashed line).

CONCLUSION

In this article, theoretical investigation of the influence of multiple beam interference on the phase retardation of liquid crystal cell is performed. As a result it is demonstrated that interference effects within the whole liquid crystal cell multilayered structure significantly affect the transmitted light phase retardation in case of monochromatic light with a very high coherence length. The high frequency of oscillations has its origin in multiple reflections at the various interfaces. Also it is shown that the frequency and amplitude of the oscillations of the phase retardation strongly depend on geometrical and optical parameters of the layers of the liquid crystal cell. For practical applications, e.g. for preparing $\lambda/4$ or $\lambda/2$ cells, the presented procedure allows to find out the parameter set, where a compensation or at least a diminishing of the influence of multiple beam interference on phase retardation of the LC cell takes place. It is also worthy to mention that the fast spectral oscillations can be observed only in case when a) the incident light has very narrow spectral width (much less then the oscillation periodicity) and b) when the all layers are strictly parallel. To make this effect more pronounce, one should utilize the conducting layers with higher refractive index (e.g. silver).

REFERENCES

[1] Holmes, D. A. (1964). *Journal of Optical Society of America*, *54*, 1116.
[2] Clarke, D. (2004). *Journal of optics A: Pure and Applied Optics*, *6*, 1036.
[3] Bibikova, E. A., Kundikova, N. D., & Rogacheva, L. F. (2003). *Proceedings of the Chelyabinsk Scientific Center*, *1*, 8.
[4] Marquez, A., Moreno, I., Campos, J., & Yzuel, M. J. (2006). *Optics Communications*, *265*, 84.
[5] Gevorgyan, A. A. (2000). *Technical Physics*, *45*(9), 1170.
[6] Jang, Y. K. & Bos, P. (2007). *Journal of applied physics*, *101*, 033131.
[7] Akahane, T. (1998). *Japanese journal of applied physics*, *37*(1–6A), 3428.
[8] Hung, L. T., Kimura, M., & Akahane, T. (2005). *Japanese Journal of Applied Physics*, *44*(2), 932.
[9] Kliger, D. S., Lewis, J. W., & Randall, C. E. (1990). *Polarized Light in Optics and Spectroscopy*, Academic Press: San Diego, California.
[10] Berreman, D. W. (1972). *Journal of Optical Society of America*, *62*, 502.
[11] Berreman, D. W. (1973). *Journal of Optical Society of America*, *63*, 1374.
[12] Palto, S. (2001). *Journal of Experimental and Theoretical Physics*, *92*(4), 552.
[13] Clarke, D. (2004). *Journal of Optics A: Pure and Applied Optics*, *62*, 1041.
[14] Heavens, O. S. (1970). *Thin Film Physics*, Methuen & Co Ltd: London.
[15] Gauza, S., Wen, C. H., Wu, S. T., Janarthanan, N., & Hsu, C. S. (2004). *Japanese Journal of Applied Physics*, *43*(11 A), 7634.
[16] Bryce, R. M., Nguyen, H. T., Nakeeran, P., Clement, T., Haugen, C. J., Tykwinski, R. R., DeCorby, R. G., & McMullin, J. N. (2004). *Thin Solid Films*, *458*, 233.
[17] Khodorov, A., Piechowiak, M., Gomes, M. J. M. (2007). *Thin Solid Films*, *515*, 7829.
[18] Wu, S. T. (1991). *Journal of Applied Physics*, *69*, 2080.